

Worksheet for 2021-11-15

Conceptual questions

Question 1. Show that the vector field $\langle y^2, xy \rangle$ is tangent to the hyperbola $y^2 - x^2 = 1$ at all points along the hyperbola.

Question 2. Find a Cartesian equation for the parametric surface $x = u + v, y = -u, z = v$ and describe its shape.

Question 3. Find a Cartesian equation for the parametric surface $x = u^2, y = -u, z = v$ and describe its shape.

Computations

Although the problems ask you to compute, the more instructive part is setting up the appropriate integral in the parameter plane.

Problem 1 (Stewart §16.7.11). Compute $\iint_S x \, dS$ if S is the triangular region with vertices $(1, 0, 0), (0, -2, 0), (0, 0, 4)$.

Problem 2 (Stewart §16.7.23). Compute $\iint_S \langle xy, yz, xz \rangle \cdot d\mathbf{S}$ if S is the part of the paraboloid $z = 4 - x^2 - y^2$ lying over the square $0 \leq x \leq 1, 0 \leq y \leq 1$, and oriented upwards.